

$$2) g(x) = x - \sqrt{x+1}$$

Df: il faut que  $x+1 \geq 0$  donc  $x \geq -1$

$$\text{Donc } Df = [-1; +\infty[$$

$$\lim_{x \rightarrow -1} g(x) = -1$$

$$\lim_{x \rightarrow +\infty} g(x) = \lim_{x \rightarrow +\infty} x \left(1 - \frac{\sqrt{x+1}}{x}\right) = \lim_{x \rightarrow +\infty} x \left(1 - \sqrt{\frac{x+1}{x^2}}\right) = \lim_{x \rightarrow +\infty} x \left(1 - \frac{1}{\sqrt{x}}\right) = +\infty$$

$$3) h(x) = \frac{\sqrt{x+3} - 2}{x-1}$$

Df: il faut que  $x-1 \neq 0$  et  $x+3 \geq 0$

Donc  $x \neq 1$  et  $x \geq -3$

$$\text{Donc } Df = [-3, 1[ \cup ]1; +\infty[$$

$$\lim_{x \rightarrow -3} h(x) = \frac{-2}{-4} = 1/2 ; \quad \lim_{x \rightarrow +\infty} h(x) = \lim_{x \rightarrow +\infty} \frac{\sqrt{x+3}}{x-1} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{x} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}} = 0$$

$$\lim_{x \rightarrow 1^+} h(x) = \lim_{x \rightarrow 1^+} \frac{(\sqrt{x+3} - 2)(\sqrt{x+3} + 2)}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \rightarrow 1^+} \frac{x+3-4}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \rightarrow 1^+} \frac{x-1}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x+3} + 2} = \frac{1}{2+2} = 1/4$$

$$\lim_{x \rightarrow 1^-} h(x) = \lim_{x \rightarrow 1^-} h(x) = 1/4$$

$$4) i(x) = \frac{x^2 \cos x + 1}{x^2 + 2}$$

Df: il faut que  $x^2 + 2 \neq 0 \Rightarrow x^2 \neq -2 \Rightarrow x \neq \sqrt[3]{-2} = d_1$   
 $x = -1, 26$

$$\text{Donc } Df = \mathbb{R} - \left\{ \sqrt[3]{-2} \right\}$$

$$\lim_{x \rightarrow +\infty} i(x) = \lim_{x \rightarrow +\infty} \frac{x^2}{x^2} = \lim_{x \rightarrow +\infty} \frac{1}{1} = 1 = \lim_{x \rightarrow -\infty} i(x)$$

$$\left[ d_1^2 \text{ ou } d_1 + 1 \neq 0, \text{ etc.} \right]$$

$$\lim_{x \rightarrow \sqrt[3]{-2}^+} i(x) = \lim_{x \rightarrow \sqrt[3]{-2}^+} \frac{x^2 \cos x + 1}{x^2 + 2} = \frac{d_1^2 \cos d_1 + 1}{0^+} = +\infty \quad \lim_{x \rightarrow \sqrt[3]{-2}^-} i(x) = -\infty$$



$$5) f(x) = \sqrt{\frac{x-1}{x^2-4}}$$

Df: il faut que  $\frac{x-1}{x^2-4} > 0 \Leftrightarrow \frac{x-1}{(x-2)(x+2)} > 0$

et  $x^2-4 \neq 0$

x	$-\infty$	$-2$	$1$	$2$	$+\infty$
$x-1$	-	-	0	+	+
$x-2$	-	-	-	-	+
$x+2$	-	-	+	+	+
$\frac{x-1}{x^2-4}$	-	+	-	+	+

$$\text{Df} = ]-2, 1[ \cup ]2, +\infty[$$

$$\lim_{x \rightarrow -2^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 1} f(x) = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = +\infty$$

$$\lim_{x \rightarrow +\infty} f(x) = 0$$

Exercice 2

$$f(x) = x + \sqrt{x^2 + x + 1}$$

$$1) \text{Df} = \{x \mid x^2 + x + 1 > 0\}$$

$$x^2 + x + 1 \quad \Delta = b^2 - 4ac = 1^2 - 4 = -3 < 0$$

De  $x^2 + x + 1$  est du signe de  $a$ , donc  $x^2 + x + 1$  est toujours positif, donc  $\text{Df} = \mathbb{R}$

$$2) \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{(x + \sqrt{x^2 + x + 1})(x - \sqrt{x^2 + x + 1})}{x - \sqrt{x^2 + x + 1}}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2 - x^2 - x - 1}{x - \sqrt{x^2 + x + 1}} = \lim_{x \rightarrow -\infty} \frac{-x - 1}{x - \sqrt{x^2 + x + 1}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x - 1}{x - \sqrt{x^2}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x - 1}{2x}$$

$$= -1/2$$

Donc  $\lim_{x \rightarrow -\infty} f(x) = -1/2$

~~$\lim_{x \rightarrow -\infty} f(x) + 1/2 = \lim_{x \rightarrow -\infty} \frac{3/2x + 1/2x^2}{x^2 + 1}$~~

Donc  $f$  admet une asymptote horizontale d'équation  $y = -1/2$

3) Plus de détails

	+	+	-	-
	+	-	-	-
	+	+	+	-
	+	-	+	-

*[Faint handwritten notes and calculations, including a table with signs and various mathematical expressions.]*