B) $I_n = \int_1^e (\ln x)^n \, dx$ ①

1)

$I_1 = \int_1^e (\ln x) \, dx$

On pose $u = \ln x$ et $v' = 1$, donc $u' = \frac{1}{x}$ et $v = x$.

$\int uv' = [uv] - \int u'v$

Donc $\int_1^e \ln(x) dx = [x \ln x]_1^e - \int_1^e \frac{1}{x} \times x \, dx = [x \ln x]_1^e - \int_1^e dx$

$= e - e + 1 = \underline{\underline{1}}$

2) $I_{n+1} = \int_1^e (\ln x)^{n+1} \, dx$

on pose $u = (\ln x)^{n+1}$ et $v' = 1$, donc $u' = (n+1)(\ln x)^n \times \frac{1}{x}$, $v = x$

Donc $\int_1^e (\ln x)^{n+1} = I_{n+1} = [(\ln x)^{n+1} \times x]_1^e - \int_1^e (n+1)(\ln x)^n \times \frac{1}{x} \times x \, dx$

$\Leftrightarrow I_{n+1} = e - \int_1^e (n+1)(\ln x)^n \, dx = e - (n+1) \int_1^e (\ln x)^n dx = \underline{\underline{e - (n+1) I_n}}$

<u>cqfd</u>

C) $I(r,s) = \int_0^1 t^r (1-t)^s \, dt$ avec $r \neq -1$ et $s \neq -1$

1) $I(r,0) = \int_0^1 t^r \, dt = \left[\frac{1}{r+1} t^{r+1}\right]_0^1 = \underline{\underline{\frac{1}{r+1}}}$

2) $I(r,s) = \int_0^1 t^r (1-t)^s \, dt = \int_1^0 (1-x)^r x^s \times (-dx)$

$= \int_0^1 (1-x)^r x^s \, dx = \int_0^1 x^s (1-x)^r \, dx = I(s,r)$.

3) $I(r,s+1) = \int_0^1 t^r (1-t)^{s+1} \, dt$

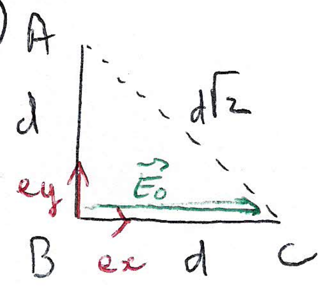
on pose $u' = t^r$ $v = (1-t)^{s+1}$, donc $u = \frac{1}{r+1} t^{r+1}$ et $v' = -(s+1)(1-t)^s$

Donc $I(r,s+1) = \int_0^1 t^r (1-t)^{s+1} dt = \left[\frac{1}{r+1} t^{r+1} \times (1-t)^{s+1}\right]_0^1 + \int_0^1 \frac{(s+1)}{(r+1)} t^{r+1} (1-t)^s dt$

Donc $I(r,s+1) = 0 + \frac{(s+1)}{(r+1)} \int_0^1 t^{r+1} (1-t)^s \, dt$

Donc $\boxed{I(r,s+1) = \frac{s+1}{r+1} I(r+1,s)}$

Exercice 3 , 1)

a) 

b) $\Delta U_{BC} = \int_C^B \vec{E_0} \cdot \vec{dl} = \int_C^B E_0 \vec{e_x} \cdot dL \vec{e_x} = \int_C^B E_0 \cdot dL$

$= -\int_B^C E_0 \cdot dL = -E_0 \int_B^C dL = \underline{\underline{-E_0 d}}$

$\Delta U_{CA} = \int_A^C \vec{E_0} \cdot \vec{dl} = \int_A^C E_0 \vec{e_x} \cdot \left(-\cos\frac{\pi}{4} \vec{e_x} + \sin\frac{\pi}{4} \vec{e_y}\right) dL$

$= \int_A^C E_0 \times \frac{-\sqrt{2}}{2} dL$

$= \int_A^C E_0 \times \frac{-\sqrt{2}}{2} \times dL = -E_0 \frac{\sqrt{2}}{2} \times d\sqrt{2} = \underline{\underline{-E_0 d}}$

$\Delta U_{AB} = \int_B^A E_0 \vec{e_x} \cdot dL \vec{e_y} = 0$ (car $\vec{e_x}$ et $\vec{e_y}$ sont $\perp$).

c) $e = \int_A^C \vec{E} \cdot \vec{dl} = E_0 d$.

2) $\vec{E} = (x^2 - y^2) \vec{e_x} - 2xy \vec{e_y}$

a) $\vec{E}(0,0) = \vec{0}$, $\underline{\vec{E}(1,0) = \vec{e_x}}$, $\underline{\vec{E}(0,1) = -\vec{e_x}}$ $\underline{\vec{E}(1,1) = -2\vec{e_y}}$

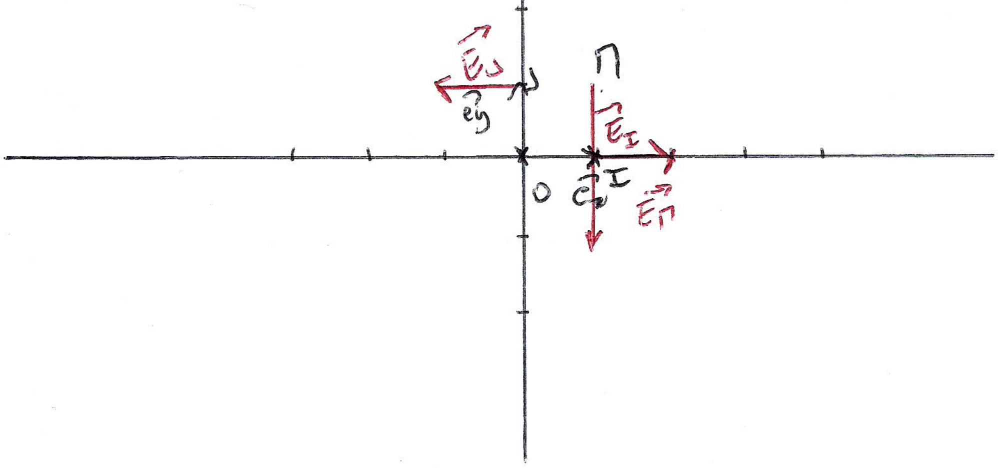
②

a) en O, $\vec{E} = \vec{0}$

en I, $\vec{E} = \vec{e_x}$

en J, $\vec{E} = -\vec{e_x}$

en M, $\vec{E} = -2\vec{e_y}$



b) Sur un contour fermé, la circulat° d'un champ électrostatique est nulle.