$\vec{AL} = a\,\vec{AD}$ $\quad\vec{AM} = a\,\vec{AB}$ $\quad\vec{CN} = a\,\vec{CG}$

1 a) $\vec{EM}\cdot\vec{EL} = (\vec{EA}+\vec{AM})\cdot(\vec{EA}+\vec{AL}) = \|\vec{EA}\|^2 + \vec{EA}\cdot\vec{AL} + \vec{AM}\cdot\vec{EA} + \vec{AM}\cdot\vec{AL}$

$= 1 + a\,\vec{EA}\cdot\vec{AD} + a\,\vec{AB}\cdot\vec{EA} + a^2\,\vec{AD}\cdot\vec{AB}$

$= 1 + 0 + 0 + 0 \quad = 1$

b) $\vec{EM}\cdot\vec{EL} = \|\vec{EM}\|\,\|\vec{EL}\|\cos(\vec{EM},\vec{EL}) = \|\vec{EM}\|\|\vec{EL}\|\cos(\widehat{MEL})$

$\|\vec{EM}\| = \|\vec{EA}+\vec{AM}\| = \|a\vec{AB} - \vec{AE}\| = \sqrt{a^2+1}$

$\|\vec{EL}\| = \|\vec{EA}+\vec{AL}\| = \|a\vec{AD} - \vec{AE}\| = \sqrt{a^2+1}.$

Donc $\vec{EM}\cdot\vec{EL} = \|\vec{EM}\|\,\|\vec{EL}\|\cos(\widehat{MEL}) = 1 = (a^2+1)\cos(\widehat{MEL})$.

Donc $\boxed{\cos(\widehat{MEL}) = \dfrac{1}{a^2+1}}$

c) $\sin^2(\widehat{MEL}) + \cos^2(\widehat{MEL}) = 1 \iff \sin^2(\widehat{MEL}) = 1 - \cos^2(\widehat{MEL})$

$\iff \sin^2(\widehat{MEL}) = 1 - \dfrac{1}{(a^2+1)^2} = \dfrac{(a^2+1)^2-1}{(a^2+1)^2} = \dfrac{a^4+2a^2+1-1}{(a^2+1)^2}$

$\iff \sin^2(\widehat{MEL}) = \dfrac{a^4+2a^2}{(a^2+1)^2} = \dfrac{a^2(a^2+2)}{(a^2+1)^2}$

Donc $\sin(\widehat{MEL}) = \sqrt{\dfrac{a^2(a^2+2)}{(a^2+1)^2}} = \dfrac{a\sqrt{a^2+2}}{a^2+1}$

d) Aire du Triangle ELM $\cancel{\dfrac{EL\times EM}{2}}$ $\cancel{\dfrac{\sqrt{a^2+1}}{2}}$ $= \|\vec{EL}\|\,\|\vec{EM}\| \times \sin(\widehat{MEL})$

$= (a^2+1) \times \dfrac{a\sqrt{a^2+2}}{a^2+1}$

$= \boxed{a\sqrt{a^2+2}}$

e) $\vec{AN}\cdot\vec{EM} = (\vec{AB}+\vec{BC}+\vec{CN})\cdot(\vec{EA}+\vec{AM})$

$= \vec{AB}\cdot\vec{EA} + \vec{AB}\cdot\vec{AM} + \vec{BC}\cdot\vec{EA} + \vec{BC}\cdot\vec{AM} + \vec{CN}\cdot\vec{EA} + \vec{CN}\cdot\vec{AM}$

$= 0 + 0 + 0 + 0 + 0 + \vec{CN}\cdot\vec{EA} + \vec{AB}\cdot\vec{AM}$

$= -a + a \quad = 0$

Donc la droite (AN) est orthogonale aux droites (EM) et (EL)