$$U_n = \int_0^{\pi} \frac{\cos nx}{(5/4 - \cos x)} \, dx$$

Donc $U_{n+1} = \int_0^{\pi} \frac{\cos(n+1)x \, dx}{(5/4 - \cos x)} = \int_0^{\pi} \frac{\cos(nx)\cos(x) - \sin(nx)\sin x}{(5/4 - \cos x)} \, dx$

$$= \int_0^{\pi} \frac{\cos nx \cos x}{(5/4 - \cos x)} - \int_0^{\pi} \frac{\sin(nx)\sin x}{(5/4 - \cos x)}$$

$$= \int_0^{\pi} \frac{\cos nx (\cos x - 5/4)}{(5/4 - \cos x)} \, dx + \int_0^{\pi} \frac{5/4 \cos nx}{(5/4 - \cos x)} \, dx$$

$$- \frac{1}{2} \int_0^{\pi} \frac{\cos(n-1)x - \cos(n+1)x}{(5/4 - \cos x)} \, dx$$

Donc $U_{n+1} = \int_0^{\pi} -\cos nx \, dx + 5/4 \, U_n - 1/2 \, U_{n-1} + \frac{1}{2} U_{n+1}$

$$U_{n+1} = \left[-\frac{1}{n} \sin nx \right]_0^{\pi} + 5/4 \, U_n - 1/2 \, U_{n-1} + 1/2 \, U_{n+1}$$

$\Leftrightarrow$ $U_{n+1} = 0 + 5/4 \, U_n - 1/2 \, U_{n-1} + 1/2 \, U_{n+1}$

$\Leftrightarrow$ $\boxed{U_{n+1} = \frac{5}{2} U_n - U_{n-1}}$

Il s'agit d'une suite de Fibonacci généralisée (ou une suite de Lucas..).