

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - x$$

$$= \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x + 1} - x \right) \left(\sqrt{x^2 + x + 1} + x \right)$$

$$\frac{\sqrt{x^2 + x + 1} - x}{\sqrt{x^2 + x + 1} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + x + 1 - x^2}{\sqrt{x^2 + x + 1} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x + 1}{\sqrt{x^2 + x + 1} + x}$$

$$\frac{x + 1}{x \left(\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1 \right)}$$

$$= \lim_{x \rightarrow \infty} \frac{x + 1}{x \left(\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1 \right)}$$

$$= \lim_{x \rightarrow \infty} \frac{1 + 1/x}{\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1}$$

$$= \frac{1}{2}$$