

Exercice 1

racines des polynômes

1) $P(x) = 2x - 1$

racine = $1/2$

2) $P(x) = -4x + 10$

racine = $\frac{10}{4} = 5/2$

3) $P(x) = x^2 - x + 4$

$\Delta = 1 - 4 \times 2 = -7$ \Rightarrow Il n'y a pas de racine

4) $P(x) = 9x^2 + x - 2$

$\Delta = 1 + 4 \times 2 \times 9 = 1 + 72 = 73$

Donc racine 1 = $\frac{-1 + \sqrt{73}}{18}$

et racine 2 = $\frac{-1 - \sqrt{73}}{18}$

Exercice 2

$$\frac{x}{x-1} + \frac{x+1}{x} = \frac{x^2 - 6x - 10}{x(x-1)}$$

$$\Leftrightarrow \frac{x^2 + (x+1)(x-1)}{x(x-1)} = \frac{x^2 - 6x - 10}{x(x-1)}$$

$$\Leftrightarrow x^2 + x^2 - 1 = x^2 - 6x - 10 \Leftrightarrow x^2 + 6x + 9 = 0 \Leftrightarrow (x+3)^2 = 0$$

$$\Leftrightarrow \boxed{x = -3}$$

Exercice 3

$$P(x) = x^3 + \frac{3}{2}x^2 - 4x + \frac{3}{2}$$

1) $P(1/2) = (1/2)^3 + \frac{3}{2}(1/2)^2 - 4 \times (1/2) + 3/2 = \frac{1}{8} + \frac{3}{8} - 2 + \frac{3}{2} = \frac{1+3-8+12}{8} = 0$

Donc $1/2$ est une racine de P

2) $\exists a, b, c \quad P(x) = (x - 1/2)(ax^2 + bx + c)$

$a = 1 \quad P(x) = (x - 1/2)(x^2 + bx + c) = x^3 + bx^2 + cx - \frac{x^2}{2} - \frac{bx}{2} - \frac{c}{2}$

$-\frac{c}{2} = \frac{3}{2} \Leftrightarrow c = -3 \quad b - 1/2 = 3/2 \Leftrightarrow b = 2$

Donc $P(x) = (x - 1/2)(x^2 + 2x - 3) = (x - 1/2)(x^2 + 2x - 3)$

$P(x) = 0 \Leftrightarrow x - 1/2 = 0$ ou $x^2 + 2x - 3 = 0$

$\Leftrightarrow x = 1/2$

ou $\Delta = 4 + 4 \times 3 = 16$

$x = \frac{-2 + \sqrt{16}}{2} = 1$

ou $x = \frac{-2 - \sqrt{16}}{2} = -3$

les solutions de $P(x) = 0$ sont $1/2$, 1 et -3 (2)

$$\begin{aligned} 3) \quad P(x) &= (x - 1/2)(x^2 + 2x - 3) \\ &= (x - 1/2)(x - 1)(x + 3) \end{aligned}$$