Exercice 1

(1)

$z = a + ib$

$(i-2)z - (2+i)\bar{z} + 6 = 0$ et $|z|^2 = b^2 + 4$

$\Leftrightarrow \begin{cases} (i-2)(a+ib) - (2+i)(a-ib) + 6 = 0 \\ a^2 + b^2 = b^2 + 4 \end{cases}$

$\Leftrightarrow \begin{cases} a^2 = 4 \\ ai - b - 2a - 2ib - (2a - 2ib + ia + b) + 6 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} a^2 = 4 \\ \cancel{ai} - b - 2a - \cancel{2ib} - 2a + \cancel{2ib} - \cancel{ia} - b + 6 = 0 \end{cases} \Leftrightarrow \begin{cases} -2b - 4a + 6 = 0 \\ a^2 = 4 \end{cases}$

$\Leftrightarrow \begin{cases} a^2 = 4 \\ b = \frac{6-4a}{2} \end{cases} \Leftrightarrow \begin{cases} a = 2 \text{ et } b = -1 \\ \text{ou} \\ a = -2 \text{ et } b = 7 \end{cases}$

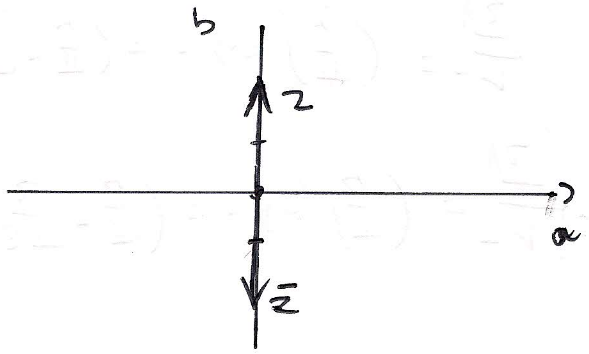
Donc $z = 2 - i$ ou $z = -2 + 7i$

Exercice 2

1) $z = \frac{(3+i)\times(1+i)}{(2-i)} = \frac{3+3i+i-1}{(2-i)} = \frac{2+4i}{(2-i)} = \frac{(2+4i)(2+i)}{5}$

$= \frac{4+2i+8i-4}{5} = 2i = 2 \times i = 2 \times e^{i\pi/2}$

Module de $z = 2$; Argument : $\pi/2$



Exercice 3

$z = 4\cos\frac{\pi}{4} + 4i\sin\frac{\pi}{4}$

$z = 4 \times \frac{1}{\sqrt{2}} + 4 \times \frac{1}{\sqrt{2}} i = 2\sqrt{2} + 2\sqrt{2}i$

Donc $z(2\sqrt{2} ; 2\sqrt{2})$

$|z| = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{8+8} = \sqrt{16} = 4.$

$\bar{z} = (2\sqrt{2} ; -2\sqrt{2})$