

Exercice 1

1) $\lim_{x \rightarrow 0} \sin(x) = \sin(0) = \underline{0}$; $\lim_{x \rightarrow +\infty} \sin(x)$ n'existe pas ; $\lim_{x \rightarrow -\infty} \sin(x)$ n'existe pas non plus.

2) $\lim_{x \rightarrow 0^+} \frac{3x^2-1}{x} = \underline{-\infty}$; $\lim_{x \rightarrow 0^-} \frac{3x^2-1}{x} = \underline{+\infty}$; $\lim_{x \rightarrow +\infty} \frac{3x^2-1}{x} = \lim_{x \rightarrow +\infty} \frac{x^2(3-1/x^2)}{x} = \lim_{x \rightarrow +\infty} x(3-1/x^2) = \underline{+\infty}$; $\lim_{x \rightarrow -\infty} \frac{3x^2-1}{x} = \underline{-\infty}$

3) $\lim_{x \rightarrow 0} x + \frac{1}{x^2} = \underline{+\infty}$; $\lim_{x \rightarrow +\infty} x + \frac{1}{x^2} = \underline{+\infty}$; $\lim_{x \rightarrow -\infty} x + \frac{1}{x^2} = \underline{-\infty}$

4) $\lim_{x \rightarrow 0^+} \frac{2x-3}{\sqrt{x}} = \underline{-\infty}$; $\lim_{x \rightarrow +\infty} \frac{2x-3}{\sqrt{x}} = \lim_{x \rightarrow +\infty} x \frac{(2-3/x)}{\sqrt{x}} = \lim_{x \rightarrow +\infty} \sqrt{x} (2-3/x) = \underline{+\infty}$.
 la limite en $-\infty$ de $\frac{2x-3}{\sqrt{x}}$ n'existe pas car $D_f =]0; +\infty[$.

5) $\lim_{x \rightarrow 0^+} \frac{2x-1}{x} = \underline{-\infty}$; $\lim_{x \rightarrow 0^-} \frac{2x-1}{x} = \underline{+\infty}$; $\lim_{x \rightarrow +\infty} \frac{2x-1}{x} = \lim_{x \rightarrow +\infty} \frac{x(2-1/x)}{x} = \lim_{x \rightarrow +\infty} 2 - \frac{1}{x} = \underline{2}$; $\lim_{x \rightarrow -\infty} \frac{2x-1}{x} = \underline{2}$

6) $\lim_{x \rightarrow 0} \frac{2x-1}{x} \sin x = \lim_{x \rightarrow 0} (2x-1) + \frac{\sin x}{x} = (-1) + 1 = \underline{-1}$; $\lim_{x \rightarrow +\infty} \frac{2x-1}{x} \sin x = \lim_{x \rightarrow +\infty} 2 \sin x$ n'existe pas ; $\lim_{x \rightarrow -\infty} \frac{2x-1}{x} \sin x$ n'existe pas non plus.

7) $\lim_{x \rightarrow 0} \sqrt{x^2+1} - x = \underline{1}$; $\lim_{x \rightarrow -\infty} \sqrt{x^2+1} - x = +\infty - (-\infty) = \underline{+\infty}$
 $\lim_{x \rightarrow +\infty} \sqrt{x^2+1} - x = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x)}{(\sqrt{x^2+1} + x)} = \lim_{x \rightarrow +\infty} \frac{x^2+1 - x^2}{(\sqrt{x^2+1} + x)} = \lim_{x \rightarrow +\infty} \frac{1}{(\sqrt{x^2+1} + x)} = \underline{0}$

8) $\lim_{x \rightarrow 0} \sqrt{x^2+2x+1} + x + 2 = \underline{3}$; $\lim_{x \rightarrow +\infty} \sqrt{x^2+2x+1} + x + 2 = \underline{+\infty}$
 $\lim_{x \rightarrow -\infty} \sqrt{x^2+2x+1} + (x+2) = \lim_{x \rightarrow -\infty} \frac{(\sqrt{x^2+2x+1} + (x+2))(\sqrt{x^2+2x+1} - (x+2))}{(\sqrt{x^2+2x+1} - (x+2))} = \lim_{x \rightarrow -\infty} \frac{(x^2+2x+1) - (x^2+4x+4)}{(\sqrt{x^2+2x+1} - (x+2))} = \lim_{x \rightarrow -\infty} \frac{-3x-3}{(\sqrt{x^2+2x+1} - (x+2))}$

