$$\vec{u} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$f(\vec{u}) = 2\vec{u} = 2\vec{u} + 0\vec{v} + 0\vec{w}$

$f(\vec{v}) = -2\vec{v} = 0\vec{u} - 2\vec{v} + 0\vec{w}$

$f(\vec{w}) = \vec{w} = 0\vec{u} + 0\vec{v} + 1\vec{w}$.

Donc $F \begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$P$ la matrice de passage de la base $\vec{i}, \vec{j}, \vec{k}$ vers $\vec{u}, \vec{v}, \vec{w}$

$$P = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$F = P^{-1} H P \iff H = P F P^{-1}$

$$\det P = 1 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = -1 - 1 = -2$$

$$P^{-1} = \begin{pmatrix} 1/2 & -1 & 1/2 \\ -1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1/2 & -1 & 1/2 \\ -1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -2 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 2 \\ 0 & 1 & 0 \\ 2 & -1 & 0 \end{pmatrix}$$