

Exercice N°1

$$\Pi_{m \times m} = \begin{pmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1m} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2m} \\ \pi_{31} & \pi_{32} & \dots & \pi_{3m} \\ \dots & \dots & \dots & \dots \\ \pi_{n1} & \pi_{n2} & \dots & \pi_{nm} \end{pmatrix}$$

$$\Pi^T = \begin{pmatrix} \pi_{11} & \pi_{21} & \pi_{31} & \dots & \pi_{n1} \\ \pi_{12} & \pi_{22} & \pi_{32} & \dots & \pi_{n2} \\ \dots & \dots & \dots & \dots & \dots \\ \pi_{1m} & \pi_{2m} & \pi_{3m} & \dots & \pi_{nm} \end{pmatrix}$$

Les éléments diagonaux de  $\Pi \Pi^T$  sont  $a_{ii} = \sum_{j=1}^m (\pi_{ij})^2$

Les éléments diagonaux de  $\Pi^T \Pi$  sont  $b_{jj} = \sum_{i=1}^m (\pi_{ji})^2$

$$\text{Re } \text{Tr}(\Pi \Pi^T) = \sum_{i=1}^m \left( \sum_{j=1}^m (\pi_{ji})^2 \right)$$

$$\text{et } \text{Tr}(\Pi^T \Pi) = \sum_{i=1}^m \left( \sum_{j=1}^m (\pi_{ij})^2 \right)$$

$$\text{Tr}(\Pi \Pi^T) = \sum_{i=1}^m \left( \sum_{j=1}^m (\pi_{ji})^2 \right) = \sum_{j=1}^m \left( \sum_{i=1}^m (\pi_{ij})^2 \right) = \text{Tr}(\Pi^T \Pi)$$

$$\underline{\underline{\text{Tr}(\Pi \Pi^T) = \text{Tr}(\Pi^T \Pi)}}$$