Question 1

1) $\cos^2\left(\frac{2\pi}{5}\right)+\sin^2\left(\frac{2\pi}{5}\right)=1 \iff \sin^2\left(\frac{2\pi}{5}\right) = 1-\cos^2\left(\frac{2\pi}{5}\right)$

Donc $\sin^2\left(\frac{2\pi}{5}\right) = 1-\left(\frac{\sqrt{5}-1}{4}\right)^2 = 1-\left(\frac{5+1-2\sqrt{5}}{16}\right) = 1-\frac{(6-2\sqrt{5})}{16}$

$= \frac{16-6+2\sqrt{5}}{16} = \frac{10+2\sqrt{5}}{16} = \frac{5+\sqrt{5}}{8}$

$0 < \frac{2\pi}{5} < \frac{\pi}{2}$, donc $\sin\left(\frac{2\pi}{5}\right) > 0$, donc $\sin\left(\frac{2\pi}{5}\right) = \sqrt{\frac{5+\sqrt{5}}{8}}$

2) $\sin(x) = \frac{1-\sqrt{5}}{4} \iff \sin(x) = -\cos\left(\frac{2\pi}{5}\right) \iff \sin(x) = \cos\left(\frac{2\pi}{5}+\pi\right)$

$\iff \sin(x) = \cos\left(\frac{7\pi}{5}\right) \iff \sin x = \sin\left(\frac{\pi}{2}-\frac{7\pi}{5}\right) = \sin\left(\frac{5\pi-14\pi}{10}\right) = \sin\left(\frac{-9\pi}{10}\right)$

$\iff \sin(x) = \sin\left(-\frac{9\pi}{10}\right) \iff \begin{cases} x = \frac{-9\pi}{10}+2k\pi \\ \text{ou} \quad\quad k\in\mathbb{Z} \\ x = \pi-\left(\frac{-9\pi}{10}\right)+2k\pi \end{cases} \iff \begin{cases} x = \frac{-9\pi}{10}+2k\pi \\ \text{ou} \\ x = \frac{19\pi}{10}+2k\pi \end{cases} k\in\mathbb{Z}.$

$\Rightarrow$ sur $[0, 2\pi]$, on obtient $x = 2\pi - \frac{9\pi}{10} = \frac{11\pi}{10}$ ou $x = \frac{19\pi}{10}$

Question 2

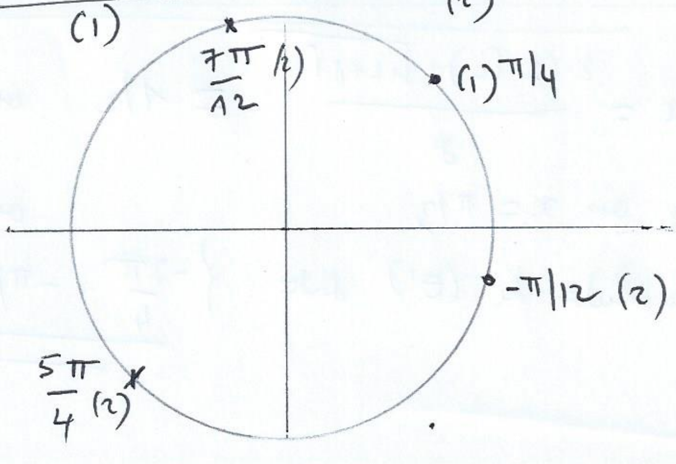
1) $\cos(2x) = \cos(x+\pi/4) \iff \begin{cases} 2x = x+\pi/4+2k\pi & k\in\mathbb{Z} \\ \text{ou} \\ 2x = -(x+\pi/4)+2k'\pi & k'\in\mathbb{Z}. \end{cases}$

$\iff \begin{cases} x = \pi/4+2k\pi & k\in\mathbb{Z} \\ \text{ou} \\ 3x = -\frac{\pi}{4}+2k'\pi & k'\in\mathbb{Z} \end{cases} \iff \begin{cases} x = \pi/4+2k\pi & k\in\mathbb{Z} \\ \text{ou} \\ x = \frac{-\pi}{12}+\frac{2k'\pi}{3} & k'\in\mathbb{Z}. \end{cases}$

Donc les solutions sont $\frac{\pi}{4}\,[2\pi]$ et $\frac{-\pi}{12}\,[2\pi/3]$.

(1) (2)

2)



Question 3

1) $2X^2 - X - 1 = 0 \quad (E_1)$

on calcule le discriminant $\Delta = (-1)^2 + 4 \times 2 = 9$

Donc $X_1 = \frac{1+3}{4} = 1$ ou $X_2 = \frac{1-3}{4} = -1/2$

Donc les solutions de (E_1) sont $\{-1/2\ ;\ 1\}$.

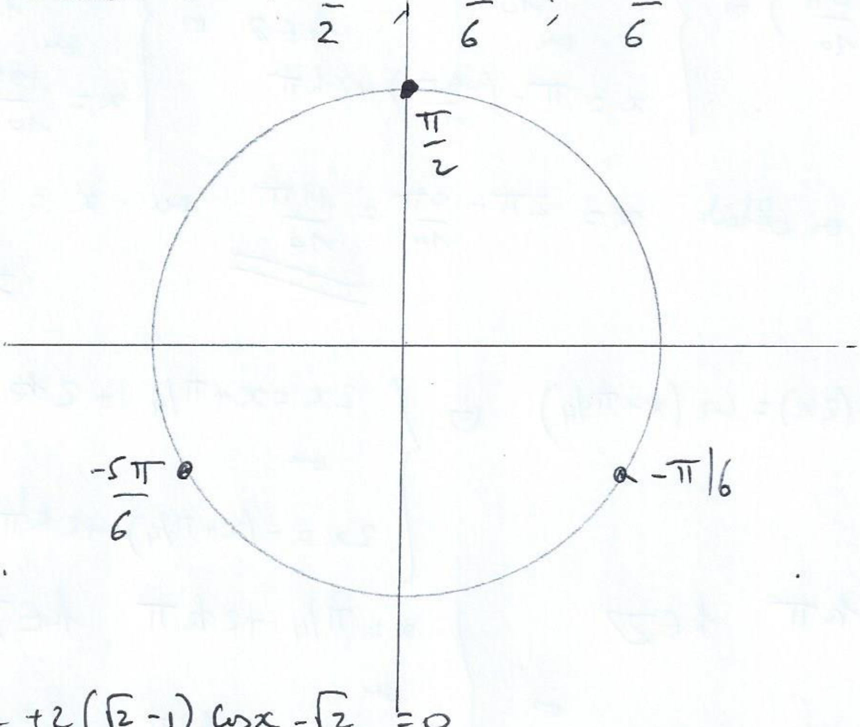
2) a) $X = \sin(x)$, donc $(E) \Leftrightarrow 2X^2 - X - 1 = 0$, on obtient l'équation du 1).

b) $2\sin^2 x - \sin(x) - 1 = 0 \Leftrightarrow \sin(x) = 1$ ou $\sin(x) = -1/2$.

$\sin(x) = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$ avec $k \in \mathbb{Z}$

$\sin(x) = -1/2 \Leftrightarrow \sin(x) = \sin(-\pi/6) \Leftrightarrow \begin{cases} x = -\pi/6 + 2k\pi \\ \text{ou} \\ x = \pi - (-\pi/6) + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = -\pi/6 + 2k\pi \\ \text{ou} \\ x = \frac{7\pi}{6} + 2k\pi. \end{cases}$

Donc sur $[-\pi ; \pi]$ les solutions sont : $\frac{\pi}{2}$; $-\frac{\pi}{6}$; $-\frac{5\pi}{6}$



3) (E') $\quad 4\cos^2 x + 2(\sqrt{2} - 1)\cos x - \sqrt{2} = 0$

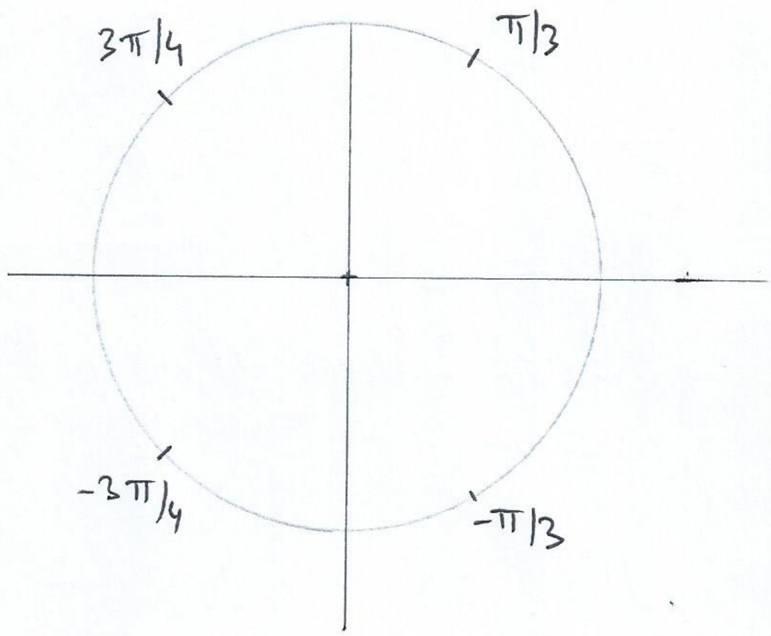
a) $\Delta = [2(\sqrt{2}-1)]^2 - 4 \times (-\sqrt{2}) \times 4 = 4(2 + 1 - 2\sqrt{2}) + 16\sqrt{2} = 12 + 8\sqrt{2} = 4(3 + 2\sqrt{2}) > 0$

Donc $X_1 = \frac{2(1-\sqrt{2}) + \sqrt{12 + 8\sqrt{2}}}{8} = \frac{1}{2}$ ou $X_2 = \frac{2(1-\sqrt{2}) - \sqrt{12+8\sqrt{2}}}{8} = -1/\sqrt{2}$

b) $(E') \Leftrightarrow \cos x = \frac{2(1-\sqrt{2}) + \sqrt{12+8\sqrt{2}}}{8} = 1/2$ ou $\cos x = -1/\sqrt{2}$

Donc $x = -\pi/3$ ou $x = \pi/3$ ou $x = \frac{3\pi}{4}$ ou $x = \frac{-3\pi}{4}$

Donc les solutions de (E') sont : $\left\{-\frac{3\pi}{4}\ ;\ -\pi/3\ ;\ \pi/3\ ;\ 3\pi/4\right\}$



4) sur $[-\pi ; \pi]$, $\sin^2 x + \sin(x) - 2 = 0$

$\Delta = 1^2 + 4 \times 2 = 9$, donc $\sin(x) = \frac{-1+3}{2}$ ou $\sin(x) = \frac{-1-3}{2}$

$\Leftrightarrow \sin(x) = 1$ ou $\sin(x) = -2$

$-1 \leq \sin(x) \leq 1$, donc $\sin(x) = -2$ est <u>IMPOSSIBLE</u>

Donc $\sin(x) = 1 \Leftrightarrow x = \pi/2 + 2k\pi$

$\boxed{\text{sur } [-\pi ; \pi] \quad x = \pi/2}$