$S_n(x) = \sum_{1}^{n} k x^{k-1}$

$f_n(x) = \sum_{0}^{n} x^k$

1) a) $S_3(1) = \sum_{1}^{3} k\, 1^{k-1} = 1+2+3 = 6$

b) $f_5(-1) = \sum_{0}^{5} (-1)^k = (-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 = 0$

2) a) $\forall x \in \mathbb{R} - \{1\}$, $f_n(x) = \sum_{0}^{n} x^k$ (f_n est la somme des termes de 0 à n de la suite geométique $u_n = (x)^n$.

Donc $f_n(x) = 1 \times \frac{1-x^{n+1}}{1-x} = \frac{x^{n+1}-1}{x-1}$ cqfd

b) $f_n(1) = \sum_{0}^{n} (1)^k = \underbrace{1+1+\cdots+1}_{n+1 \text{ fois}} = \underline{\underline{n+1}}$

3) a) $f_n(x) = 1 + x + x^2 + \cdots + x^n$

$f'_n(x) = 1 + 2x + \cdots + n x^{n-1} = \sum_{1}^{n} k x^{k-1} = S_n(x)$

Donc $S_n(x)$ est la derivée de la fonction $f_n(x)$.

b) $f_n(x) = \frac{x^{n+1}-1}{x-1}$, donc $f'_n(x) = \frac{(n+1)x^n(x-1) - (x^{n+1}-1)}{(x-1)^2}$

$= \underline{\underline{\frac{n x^{n+1} - (n+1)x^n + 1}{(x-1)^2}}} = \underline{\underline{S_n(x)}}$

4) $\sum_{1}^{100} k \times 2^{k-1} = S_{100}(2) = \frac{100 \times 2^{101} - 101 \times 2^{100} + 1}{1^2} = 2^{100}(200-101)+1$

$= \underline{\underline{99 \times 2^{100} + 1}}$ cqfd