

$$u_0 = 3$$

$$u_{n+1} = \sqrt{2 + u_n}$$

1) Montrons que $u_1 - u_0 < 0$

$$u_0 = 3 \Rightarrow u_1 = \sqrt{2+3} = \sqrt{5}$$

$$\sqrt{5} - 3 < 0$$

2) Supposons que $u_{n+2} - u_n < 0$, Montrons que

$$u_{n+2} - u_{n+1} < 0$$

$$\begin{aligned} u_{n+2} - u_{n+1} &= \sqrt{2 + u_{n+1}} - \sqrt{2 + u_n} \\ &= \frac{(\sqrt{2 + u_{n+1}} - \sqrt{2 + u_n})(\sqrt{2 + u_{n+1}} + \sqrt{2 + u_n})}{\sqrt{2 + u_{n+1}} + \sqrt{2 + u_n}} \\ &= \frac{(2 + u_{n+1}) - (2 + u_n)}{\sqrt{2 + u_{n+1}} + \sqrt{2 + u_n}} \\ &= \frac{u_{n+1} - u_n}{\sqrt{2 + u_{n+1}} + \sqrt{2 + u_n}} \rightarrow < 0 \end{aligned}$$

Donc $u_{n+2} - u_{n+1} < 0$ CQFD

Donc la suite u_n est bien décroissante.