$f(x) = x^2 - 6x - 3$

1) $(x-3)^2 - 12 = x^2 - 6x + 9 - 12$

$= x^2 - 6x - 3$

$= f(x)$

D'où $f(x) = (x-3)^2 - 12$

2) $f(x) = (x-3)^2 - 12 = (x-3)^2 - (\sqrt{12})^2$

$= (x-3)^2 - (2\sqrt{3})^2$

$= [(x-3) - 2\sqrt{3}][(x-3) + 2\sqrt{3}]$

$= (x - 3 - 2\sqrt{3})(x - 3 + 2\sqrt{3})$

3) a) $f(x) = -12 \iff (x-3)^2 - 12 = -12 \iff (x-3)^2 = 0$

$\iff x - 3 = 0 \iff \underline{x = 3}$

b) $f(x) = 0 \iff (x - 3 - 2\sqrt{3})(x - 3 + 2\sqrt{3}) = 0$

$\iff x - 3 - 2\sqrt{3} = 0$ ou $x - 3 + 2\sqrt{3} = 0$

$\iff \underline{x = 3 + 2\sqrt{3}}$ ou $\underline{x = 3 - 2\sqrt{3}}$

c) $f(x) = -6x \iff x^2 - 6x - 3 = -6x \iff x^2 - 3 = 0$

$\iff x^2 = 3 \iff \underline{x = \sqrt{3}}$ ou $\underline{x = -\sqrt{3}}$