

Exercise 3

$$a) (1+y)y' = x-1$$

$$\Leftrightarrow (1+y)dy = (x-1)dx$$

$$\Leftrightarrow \int (1+y)dy = \int (x-1)dx$$

$$\Leftrightarrow y + \frac{y^2}{2} = \frac{x^2}{2} - x + C_1 + C_2$$

$$\Leftrightarrow 2y + y^2 = x^2 - 2x + C_1 + C_2$$

$$\Leftrightarrow y^2 + 2y + 1 = x^2 - 2x + 1 + C_1 + C_2$$

$$y+1 = \pm \sqrt{(x-1)^2 + C_1 + C_2}$$

$$b) 2y' = \frac{y}{x} + \frac{y^2}{x^2}$$

$$\Leftrightarrow \text{pde } u = \frac{1}{y} \quad (\text{avec } y = \frac{1}{u})$$

$$-2\frac{u'}{u} = \frac{1}{ux} + \frac{1}{u^2x^2}$$

$$\Leftrightarrow -2u'u = \frac{1}{x} + \frac{1}{x^2}$$