Exercice 2

$AB = c = 6 \qquad AC = b = 5 \qquad BC = a = 7$

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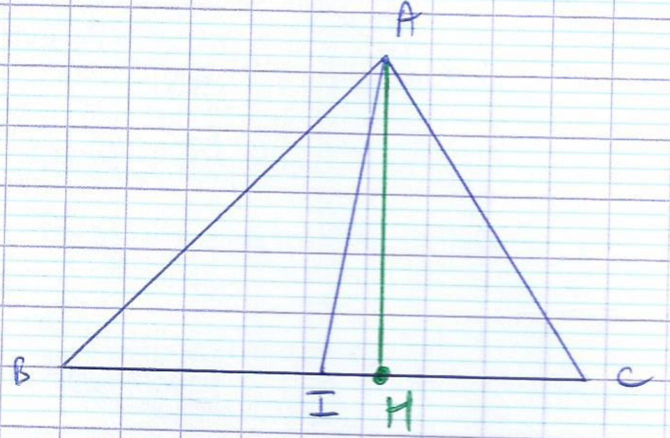

D'après la formule dans le triangle ABC

$$AC^2 + AB^2 = 2AI^2 + \frac{BC^2}{2} \Rightarrow AI^2 = \frac{AC^2}{2} + \frac{AB^2}{2} - \frac{BC^2}{4}$$

$$\Rightarrow AI^2 = \frac{5^2}{2} + \frac{6^2}{2} - \frac{7^2}{4} = 12,5 + 18 - 12,25 = 18,25$$

$$\Rightarrow AI = \sqrt{18,25}$$

2 D'après Al Kashi

$$AC^2 = AB^2 + BC^2 - 2AB \times BC \times \cos\hat{\beta}$$

Donc $\cos\hat{\beta} = \dfrac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC} = \dfrac{6^2 + 7^2 - 5^2}{2 \times 6 \times 7}$

$$= \frac{60}{84} = \frac{30}{42} = \frac{15}{21} = \frac{5}{7}$$

$\Rightarrow$ [illegible] $\qquad \Rightarrow \boxed{\hat{\beta} \approx 44,4°}$

3) $\cos\hat{\beta} = \frac{5}{7} \Rightarrow \sin\hat{\beta} = \sqrt{1 - \frac{25}{49}} = \sqrt{\frac{24}{49}} = \frac{2\sqrt{6}}{7}$

D'autre part, $\sin\hat{\beta} = \frac{AM}{AB}$

$\Rightarrow \frac{2\sqrt{6}}{7} = \frac{AM}{6} \Leftrightarrow AM = \frac{2\times 6\sqrt{6}}{7} = \frac{12\sqrt{6}}{7}$ CQFD

4) $\text{Aire }(ABC) = S = \frac{BC \times AM}{2} = \frac{7 \times 12\sqrt{6}}{7\times 2} = \underline{\underline{6\sqrt{6}}}\ \text{cm}^2$

5) Soit O le centre du cercle inscrit.

$S = \text{Aire}(BOC) + \text{Aire}(BOA) + \text{Aire}(AOC)$

$= \frac{r\times BC}{2} + r\times\frac{AB}{2} + r\times\frac{AC}{2}$

$= r\frac{a}{2} + r\frac{c}{2} + r\frac{b}{2}$

$= \frac{r(a+b+c)}{2}$ CQFD.

$S = \frac{r(a+b+c)}{2} = 6\sqrt{6} \Rightarrow r = \frac{2\times 6\sqrt{6}}{a+b+c}$

$\Rightarrow r = \frac{2\times 6\times\sqrt{6}}{7+6+5} = \frac{2\times 6\times\sqrt{6}}{18} = \frac{12}{18}\sqrt{6}$

$= \frac{2}{3}\sqrt{6}$ CQFD