Exercice 4

soit a la longueur du coté des carrés.

D'après la trigonométrie du triangle rectangle.

$$\sin \alpha = \frac{CD}{AD} \quad ; \quad \cos \alpha = \frac{AC}{AD}$$

$$\sin \beta = \frac{DC}{BD} \quad ; \quad \cos \beta = \frac{BC}{BD}.$$

On a : $CD = a$, $AC = 3a$; $BC = 2a$

$$AD^2 = DC^2 + AC^2 = a^2 + 9a^2 = 10a^2 \Rightarrow AD = a\sqrt{10}$$

$$BD^2 = DC^2 + BC^2 = a^2 + 4a^2 = 5a^2 \Rightarrow BD = a\sqrt{5}$$

Donc $\sin \alpha = \dfrac{a}{a\sqrt{10}} = \dfrac{1}{\sqrt{10}}$; $\cos \alpha = \dfrac{3a}{a\sqrt{10}} = \dfrac{3}{\sqrt{10}}$

on remarque que $\alpha < \pi/4$

$\sin \beta = \dfrac{a}{a\sqrt{5}} = \dfrac{1}{\sqrt{5}}$; $\cos \beta = \dfrac{2a}{a\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

on remarque que $\beta < \pi/4$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \frac{1}{\sqrt{10}} \times \frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{5}} = \frac{2+3}{\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

$\sin(\alpha + \beta) = \dfrac{1}{\sqrt{2}}$, donc $\underline{\alpha + \beta = \pi/4}$ (car $\alpha + \beta < \pi/2$).