Demonstration par induction

$$\sum_{1}^{n} (6k^2+2k+1) = 2n^3+4n^2+3n$$

Montrons que la proposition est vraie au rang 1.

$n=1$ $\quad \sum_{1}^{1} (6k^2+2k+1) = 6+2+1 = 9$

et $\quad 2\times(1)^3+4\times(1)+3\times1 = 9$

Donc $\underline{\underline{\text{OK}}}$

Supposons que $\quad \sum_{1}^{n} (6k^2+2k+1) = 2n^3+4n^2+3n$

Montrons que $\quad \sum_{1}^{n+1} (6k^2+2k+1) = 2(n+1)^3+4(n+1)^2+3(n+1)$

$$\begin{aligned}
\sum_{1}^{n+1} (6k^2+2k+1) &= \sum_{1}^{n} (6k^2+2k+1) + \sum_{n+1}^{n+1} (6k^2+2k+1) \\
&= 2n^3+4n^2+3n + 6(n+1)^2+2(n+1)+1 \\
&= 2n^3+4n^2+3n + 6n^2+12n+6+2n+2+1 \\
&= 2n^3+10n^2+17n+9 \quad (1)
\end{aligned}$$

D'autre part,

$$\begin{aligned}
2(n+1)^3+4(n+1)^2+3(n+1) &= 2(n+1)(n^2+2n+1)+4(n^2+2n+1)+3(n+1) \\
&= 2n^3+4n^2+2n+2n^2+4n+2+4n^2+8n+4 \\
&\quad +3n+3 \\
&= 2n^3+10n^2+17n+9 \quad (2)
\end{aligned}$$

(1) et (2), donc la demonstration est faite.