$$x^3 y + x^2 y' = x+1$$

on pose $\underbrace{ux^3 y}_{v'} + \underbrace{ux^2 y'}_{v} = u(x+1)$

$v' = ux^3$ et $ux^2 = v$

Donc $ux^3 = u'x^2 + 2xu$

$\Leftrightarrow \quad u(x^3 - 2x) = u'x^2$

$\Leftrightarrow \quad \frac{u'}{u} = x - \frac{2}{x}$

$\Leftrightarrow \quad \ln(u) = \frac{x^2}{2} - 2\ln(x) + K$

$u = \alpha e^{x^2/2 - 2\ln(x)} = \alpha e^{x^2/2} + e^{2\ln(1/x)}$

$= \frac{\alpha}{x^2} e^{x^2/2}$

on a donc $\frac{\alpha}{x^2} x^3 e^{x^2/2} y + \frac{\alpha e^{x^2/2}}{x^2} x^2 y' = \frac{\alpha(x+1)}{x^2} e^{x^2/2}$

$\Leftrightarrow \quad \left[\alpha x e^{x^2/2} y + \alpha e^{x^2/2} y' \right] = \frac{\alpha(x+1)}{x^2} e^{x^2/2}$

$\Leftrightarrow \quad \left[\alpha e^{x^2/2} y \right]' = \alpha \frac{(x+1)}{x^2} e^{x^2/2}$

$\alpha e^{x^2/2} y = \int \frac{\alpha(x+1)}{x^2} e^{x^2/2} + K.$

$\Leftrightarrow \quad y = \frac{\int \frac{(x+1)}{x^2} e^{x^2/2}}{e^{x^2/2}} + \beta e^{-x^2/2}$