$$I_n = \frac{1}{2} \sum_{k=-n}^{n} \sum_{0}^{+\infty} \int_{2p\pi}^{2(p+1)\pi} e^{ikx} e^{-ax/2} \, dx .$$

$$\int_{2p\pi}^{2(p+1)\pi} e^{ikx} e^{-\frac{ax}{2}} \, dx$$

$$= \int_{2p\pi}^{2(p+1)\pi} e^{x(ik - a/2)} \, dx$$

$$= \frac{1}{(ik - a/2)} \left[e^{x(ik - a/2)} \right]_{2p\pi}^{2(p+1)\pi}$$

$$= \frac{1}{(ik - a/2)} \left[e^{ikx} e^{-ax/2} \right]_{2p\pi}^{2(p+1)\pi}$$

$$= \frac{1}{ik - a/2} \left[e^{-a(p+1)\pi} - e^{-ap\pi} \right]$$

$$= \frac{-(ik + a/2)}{(k^2 + a^2/4)} e^{-ap\pi} \left[e^{-\pi a} - 1 \right]$$

$$\sum_{0}^{+\infty} \frac{-(ik + a/2)}{k^2 + a^2/4} e^{-ap\pi} \left[e^{-\pi a} - 1 \right]$$

$$= \frac{-(ik + a/2)}{k^2 + a^2/4} \frac{1}{1 - e^{-\pi a}} \left[e^{-\pi a} - 1 \right]$$

$$= \frac{ik + a/2}{k^2 + a^2/4}$$

$$\frac{1}{2} \sum_{k=-n}^{n} \frac{ik + a/2}{k^2 + a^2/4}$$

$$= \frac{1}{2} \sum_{-n}^{n} \frac{a/2}{k^2+a^2/4} \qquad \text{car} \quad \sum_{-n}^{n} \frac{ik}{k^2+a^2/4} = 0$$

$$= \frac{1}{4} \sum_{-n}^{n} \frac{a}{k^2+a^2/4} = \frac{1}{4} \sum_{-n}^{n} \frac{a}{k^2+(a^2/4)}$$

$$\lim_{+\infty} I_n = \frac{1}{4} \sum_{-\infty}^{+\infty} \frac{a}{k^2+a^2/4}$$

$$= \frac{1}{4} \sum_{-\infty}^{+\infty} \frac{4a}{4k^2+a^2}$$

$$= \sum_{-\infty}^{+\infty} \frac{a}{4k^2+a^2}$$