Exercice III

$$f(x) = \cosh(\sqrt{2x})$$

$$\forall x \in \mathbb{R}_+^*, \quad f'(x) = \sinh(\sqrt{2x}) \times \frac{2}{2\sqrt{2x}}$$

$$= \frac{\sinh(\sqrt{2x})}{\sqrt{2x}}$$

$$= \frac{e^{\sqrt{2x}} - e^{-\sqrt{2x}}}{2\sqrt{2x}}$$

On pose $X = \sqrt{2x}$

on a : $f'(x) = \dfrac{e^{X} - e^{-X}}{2X}$

$$\lim_{x \to 0} f'(x) = \lim_{X \to 0} f'(x) = \lim_{X \to 0} \frac{e^{X} + e^{-X}}{2}$$ (Règle de l'hopital)

$$= \underline{\underline{1}}$$

Donc f est de classe $C1$ sur $\mathbb{R}^+$.

$$f''(x) = \frac{\left(\dfrac{\cosh(\sqrt{2x})}{\sqrt{2x}} + \sqrt{2x}\right) - \left(\sinh(\sqrt{2x}) + \dfrac{1}{\sqrt{2x}}\right)}{2x}$$

$$= \frac{\cosh(\sqrt{2x}) - \dfrac{\sinh(\sqrt{2x})}{\sqrt{2x}}}{2x}$$

$$= \frac{\sqrt{2x}\cosh(\sqrt{2x}) - \sinh(\sqrt{2x})}{2x\sqrt{2x}}$$

Règle de l'hopital

si $\lim_{x \to x_0} \dfrac{f(x)}{g(x)}$ = forme indéterminée $\left(\frac{0}{0} \text{ ou } \frac{\infty}{\infty}\right)$

alors

$$\lim_{x_0} \frac{f(x)}{g(x)} = \lim_{x_0} \frac{f'(x)}{g'(x)}$$

On pose $\sqrt{2x} = X$

$$f''(x) = \frac{X\cosh(X) - \sinh(X)}{X^3}$$

$$\lim_{x \to 0} f''(x) = \lim_{X \to 0} \frac{X\cosh(X) - \sinh(X)}{X^3}$$

$$= \lim_{X \to 0} \frac{\cosh(X) + X\sinh(X) - \cosh(X)}{3X^2}$$

$$= \lim_{X \to 0} \frac{X\sinh(X)}{3X^2}$$

$$= \lim_{X \to 0} \frac{\sinh(X)}{3X}$$

$$= \lim_{X \to 0} \frac{\cosh(X)}{3}$$

$$= 1/3$$

Donc f est de classe C_2