Exercice 3



1) $\int u'v = [uv] - \int uv'$

$$I = \int 2t \arcsin(t)\, dt$$

on pose $u' = 2t$ $\quad v = \arcsin(t)$ $\Rightarrow u = t^2$ et $v' = \dfrac{1}{\sqrt{1-t^2}}$

$$I = t^2 \arcsin(t) - \int \frac{t^2}{\sqrt{1-t^2}}\, dt$$

on pose $J = \displaystyle\int \frac{t^2}{\sqrt{1-t^2}}\, dt$ $\quad$ avec $t = \sin x$

Donc $J = \displaystyle\int \frac{\sin^2 x}{\cos x} \times \cos x\, dx = \int \sin^2 x\, dx$

$$= \int \frac{1-\cos 2x}{2}\, dx$$

$$= \frac{x}{2} - \frac{\sin(2x)}{4}$$

$$= \frac{x}{2} - \frac{\sin x \cos x}{2}$$

$$= \frac{\operatorname{Arcsin}(t)}{2} - \frac{t\sqrt{1-t^2}}{2}$$

$$\boxed{\text{Donc } I = t^2 \operatorname{Arcsin}(t) - \frac{\operatorname{Arcsin}(t)}{2} + \frac{t\sqrt{1-t^2}}{2} + C}$$

2) $K = \displaystyle\int \operatorname{Arcsin}(\sqrt{x})\, dx$

on pose $x = t^2$

$$I_1 = \int \mathrm{Arcsin}(\sqrt{x})\,dx = \int \mathrm{Arcsin}(t) \times 2t\,dt$$

$$= \int 2t\,\mathrm{Arcsin}(t)\,dt$$

$$= t^2\,\mathrm{Arcsin}(t) - \frac{\mathrm{Arcsin}(t)}{2} + \frac{t\sqrt{1-t^2}}{2}$$

$$= x\,\mathrm{Arcsin}(\sqrt{x}) - \frac{\mathrm{Arcsin}(\sqrt{x})}{2} + \frac{\sqrt{x}\sqrt{1-x}}{2}$$

$$\boxed{I_1 = x\,\mathrm{Arcsin}(\sqrt{x}) - \frac{\mathrm{Arcsin}(\sqrt{x})}{2} + \frac{\sqrt{x-x^2}}{2} + C}$$