$$f^{-1} \circ f(x) = x$$

$$\Leftrightarrow \ f^{-1}[f(x)] = x$$

$\Leftrightarrow$ pose $f(x) = \tan(x)$, donc $f^{-1}(x) = \text{Arctan}(x)$

$$(f \circ g)' = f'(g) + g'$$

Donc $(f^{-1} \circ f)' = (f^{-1})'(f) + f' = 1.$

$$(f^{-1})'(f) \times \cancel{1+\tan^2(x)} \ \frac{1}{\cos^2 x} = 1.$$

$$\Leftrightarrow (f^{-1})'(f(x)) = \cos^2 x = \frac{1}{1+\tan^2(x)}$$

$$\Leftrightarrow (f^{-1})'(f) = \frac{1}{1+f^2}$$

$$\Leftrightarrow (f^{-1})'(x) = \frac{1}{1+x^2}$$

$$\Rightarrow \boxed{(\text{Arctan})'(x) = \frac{1}{1+x^2} \qquad \text{CQFD}}$$