Problème 2

$$\cos^2\left(x+\frac{2\pi}{3}\right) = \sin^2\left(x+\frac{2\pi}{3}\right)$$

$$\Leftrightarrow \cos^2\left(x+\frac{2\pi}{3}\right) = \cos^2\left(\frac{\pi}{2}-\left(x+2\pi/3\right)\right)$$

$$\Leftrightarrow \cos^2\left(x+\frac{2\pi}{3}\right) = \cos^2\left(-\frac{\pi}{6}-x\right) \Leftrightarrow \cos^2\left(x+\frac{2\pi}{3}\right) = \cos^2\left(\frac{\pi}{6}+x\right)$$

$$\Leftrightarrow \begin{cases} \cos\left(x+2\pi/3\right) = \cos\left(\pi/6+x\right) & (1) \\ \text{ou} \\ \cos\left(x+\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{6}+x\right) & (2) \end{cases}$$

$$(1) \quad \cos\left(x+\frac{2\pi}{3}\right) = \cos\left(\pi/6+x\right)$$

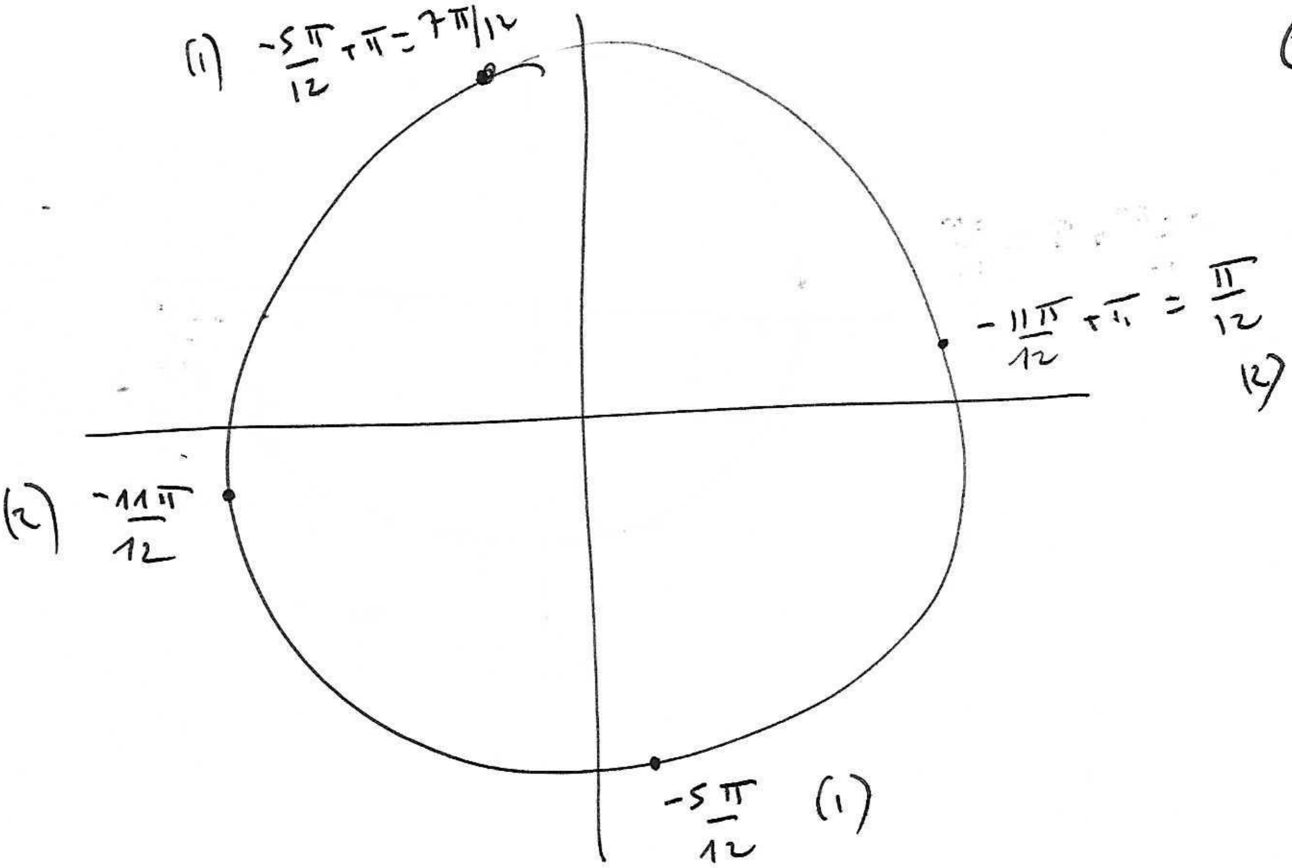
$$\Leftrightarrow \begin{cases} x+2\pi/3 = \pi/6+x+2k\pi \\ \text{ou} \\ x+\frac{2\pi}{3} = -\left(\pi/6+x\right)+2k\pi \end{cases} \Leftrightarrow \begin{cases} \text{IMPOSSIBLE} \\ \\ 2x = \frac{-2\pi}{3}-\frac{\pi}{6}+2k\pi \end{cases}$$

$$\Rightarrow \left\{ 2x = \frac{-5\pi}{6}+2k\pi \right. \quad \Leftrightarrow \left\{ x = \frac{-5\pi}{12}+k\pi \right.$$

$$(2) \Leftrightarrow \cos\left(x+\frac{2\pi}{3}\right) = \cos\left(\frac{7\pi}{6}+x\right)$$

$$\Leftrightarrow \begin{cases} x+\frac{2\pi}{3} = \frac{7\pi}{6}+x+2k\pi \\ \text{ou} \\ x+\frac{2\pi}{3} = \frac{-7\pi}{6}-x+2k\pi \end{cases} \Leftrightarrow \begin{cases} \text{IMPOSSIBLE} \\ \\ 2x = \frac{-11\pi}{6}+2k\pi \end{cases}$$

$$\Rightarrow \left\{ x = \frac{-11\pi}{12}+k\pi \right.$$



Problème N° 3

$$\operatorname{tg}(x+\pi/3) + \operatorname{cotg}\left(\frac{\pi}{2} - 3x\right) = 0$$

$$\Leftrightarrow \quad \operatorname{tg}(x+\pi/3) = \operatorname{cotg}(3x - \pi/2)$$

$$\Leftrightarrow \quad \operatorname{tg}(x+\pi/3) = \operatorname{tg}\left(\frac{\pi}{2} - (3x - \pi/2)\right)$$

$$\Leftrightarrow \quad \operatorname{tg}(x+\pi/3) = \operatorname{tg}(\pi - 3x)$$

$$\Leftrightarrow \quad \operatorname{tg}(x+\pi/3) = \operatorname{tg}(-3x) \qquad (3)$$

Il faut que $x+\pi/3 \neq \pi/2 + k\pi$ (4)

et $-3x \neq \pi/2 + k\pi$ (5)

$$(3) \Leftrightarrow \quad \frac{\sin(x+\pi/3)}{\cos(x+\pi/3)} = \frac{\sin(-3x)}{\cos(-3x)}$$

$$\Leftrightarrow \quad \sin(x+\pi/3)\cos(-3x) = \cos(x+\pi/3)\sin(-3x)$$

$$\Leftrightarrow \quad \sin(x+\pi/3)\cos(-3x) - \cos(x+\pi/3)\sin(-3x) = 0$$

$$\sin\left(x+\pi/3-(-3x)\right)=0$$

$$\Leftrightarrow \sin(4x+\pi/3)=0$$

$$\Leftrightarrow 4x+\pi/3=k\pi \qquad k\in\mathbb{Z}$$

$$\Leftrightarrow 4x=-\frac{\pi}{3}+k\pi \qquad k\in\mathbb{Z}$$

$$\Leftrightarrow x=-\frac{\pi}{12}+\frac{k\pi}{4} \qquad k\in\mathbb{Z}.$$

$$\cancel{S=\left\{-\frac{\pi}{12}+\frac{k\pi}{4} \quad k\in\mathbb{Z}\right\}}$$

(4) Il faut que $x+\frac{\pi}{3}\neq\frac{\pi}{2}+k'\pi$

$$\Leftrightarrow -\frac{\pi}{12}+\frac{k\pi}{4}+\frac{\pi}{3}\neq\frac{\pi}{2}+k'\pi$$

$$\Leftrightarrow \frac{\pi}{4}+k\frac{\pi}{4}\neq\frac{\pi}{2}+k'\pi$$

$$\Leftrightarrow \frac{\pi}{4}(1+k)\neq\pi\left(\frac{1}{2}+k'\right)$$

$$\Leftrightarrow 1+k\neq 2+4k'$$

$$\Leftrightarrow k\neq 1+4k'$$

(5) Il faut que $-3x\neq\frac{\pi}{2}+k'\pi$

$$\Leftrightarrow -3\left(-\frac{\pi}{12}+\frac{k\pi}{4}\right)\neq\frac{\pi}{2}+k'\pi$$

$$\Leftrightarrow \frac{\pi}{4}-\frac{3\pi k}{4}\neq\frac{\pi}{2}+k'\pi$$

$$\Leftrightarrow 1-3k\neq 2+4k'$$

$$\Leftrightarrow 3k\neq -1-4k'$$

Donc

$$S = \left\{ \frac{-\pi}{12} + \frac{k\pi}{4} \; ; \; k \in \mathbb{Z} \right.$$
$$\left. \text{avec } k \neq 1 + 4k' \text{ et } k \neq \frac{-1-4k'}{3} \right\}$$

Problème 4

$$-4\cos^2 x + 2(\sqrt{3}-1)\sin x + 4 - \sqrt{3} = 0 \qquad (E)$$

on pose $\cos^2 x = 1 - \sin^2 x$

$$(E) \Leftrightarrow -4(1-\sin^2 x) + 2(\sqrt{3}-1)\sin x + 4 - \sqrt{3} = 0$$
$$\Leftrightarrow -4 + 4\sin^2 x + 2(\sqrt{3}-1)\sin x + 4 - \sqrt{3} = 0$$
$$\Leftrightarrow 4\sin^2 x + 2(\sqrt{3}-1)\sin x - \sqrt{3} = 0$$

on pose $\Delta = b^2 - 4ac$

$$= \left[2(\sqrt{3}-1)\right]^2 - 4 \times 4(-\sqrt{3})$$
$$= 4(3+1-2\sqrt{3}) + 16\sqrt{3}$$
$$= 4(4-2\sqrt{3}) + 16\sqrt{3}$$
$$= 16 - 8\sqrt{3} + 16\sqrt{3}$$
$$= 16 + 8\sqrt{3}$$
$$= (2+2\sqrt{3})^2$$

Donc $\sin x = \dfrac{-2(\sqrt{3}-1) + \sqrt{16+8\sqrt{3}}}{8}$ ou $\sin x = \dfrac{-2(\sqrt{3}-1) - \sqrt{16+8\sqrt{3}}}{8}$

Rac

$\sin x = \dfrac{-2\sqrt{3}+2+2+2\sqrt{3}}{8}$ $\qquad$ $\sin x = \dfrac{-2\sqrt{3}+2-2-2\sqrt{3}}{8}$

ou

$\sin x = \dfrac{1}{2}$ $\qquad$ $\sin x = \dfrac{-\sqrt{3}}{2}$

$x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$ $\quad k \in \mathbb{Z}$

$x = -\pi/3 + 2k'\pi$ ou $x = \dfrac{4\pi}{3} + 2k'\pi$ $\quad k \in \mathbb{Z}$